Pat's score on today's midterm depends on how much sleep Pat got in the last 24 hours. **SCORE: _____ / 15 PTS**

Let $m = g(s)$, where m is the number of points Pat scores on the midterm, and s is the number of hours Pat slept in the last 24 hours.

[a] What are the units of $g'(s)$?

POINTS / HOUR

[b] What does $g'(5) = 11$ mean ? **<u>Give the units for all numbers in your answer.</u>**

IF PAT HAD SLEPT FOR 5 OF THE LAST 24 HOURS BEFORE THE MIDTERM, PAT'S SCORE WOULD HAVE INCREASED $\approx$ 11 POINTS FOR EACH ADDITIONAL HOUR OF SLEEP PAT HAD GOTTEN.

[c] Is there a value of s_0 for which you would expect $g'(s_0) < 0$? Why or why not ?

YES. IF PAT SLEPT TOO MUCH, THEN PAT STUDIED LESS OR COULD HAVE ENDED UP GROGGY, AND MORE SLEEP WOULD CAUSE A DROP IN PAT'S SCORE

Prove that the equation $x = 3\ln x$ has a solution in the interval $(1, e)$.　　**SCORE: _____ / 15 PTS**

LET $f(x) = x - 3\ln x$

f IS CONT. ON $(1,e)$ SINCE x AND $3\ln x$ ARE BOTH CONT. ON $(1,e)$

(POLY + LOG)

SO THEIR DIFFERENCE IS ALSO CONT.

$f(e) = e - 3 < 0 < 1 = f(1)$

BY IVT, THERE IS A VALUE $c \in (1,e)$

SUCH THAT $f(c) = 0$

IE. $c - 3\ln c = 0$

$c = 3\ln c$

SO $x = 3\ln x$ FOR SOME $x \in (1,e)$

State the Squeeze Theorem. **SCORE: _____ / 10 PTS**

IF $f(x) \le g(x) \le h(x)$ FOR ALL x NEAR a (EXCEPT POSSIBLY AT a)

AND $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$

THEN $\lim\limits_{x \to a} g(x) = L$

State the formal definition of "removable discontinuity". **SCORE: _____ / 5 PTS**

f HAS A REMOVABLE DISCONTINUITY AT a IF $\lim_{x \to a} f(x)$ EXISTS BUT EITHER $f(a)$ DOES NOT EXIST OR $\lim_{x \to a} f(x) \neq f(a)$

Let $f(x) = \dfrac{x-1}{x(x+3)^2}$.

SCORE: _____ / 20 PTS

[a] Find all intervals on which f is continuous.

$x \neq 0, -3$

$(-\infty, -3), (-3, 0), (0, \infty)$

[b] Find the limit of f at each discontinuity.

Each limit should be a number, ∞ or $-\infty$. **<u>Write DNE only if the other possibilities do not apply.</u>**

$\lim_{x \to 0^+} \dfrac{x-1}{x(x+3)^2} = -\infty$ $\quad \dfrac{-1}{0^+}$

$\lim_{x \to 0^-} \dfrac{x-1}{x(x+3)^2} = \infty$ $\quad \dfrac{-1}{0^-}$

$\lim_{x \to 0} f(x)$ DNE

$\lim_{x \to -3^+} \dfrac{x-1}{x(x+3)^2} = \infty$ $\quad \dfrac{-4}{0^-}$

$\lim_{x \to -3^-} \dfrac{x-1}{x(x+3)^2} = \infty$ $\quad \dfrac{-4}{0^-}$

$\lim_{x \to -3} f(x) = \infty$

Let $f(x) = \frac{1}{\sqrt{x}}$.

SCORE: _____ / 30 PTS

[a] Find $f'(x)$.

$$\lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{\cancel{x} - (\cancel{x}+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \frac{-1}{2x\sqrt{x}} \text{ OR } -\frac{1}{2}x^{-\frac{3}{2}}$$

[b] Find the equation of the tangent to the graph of f at the point where $x = 9$.

$f(9) = \frac{1}{3}$ $f'(9) = -\frac{1}{54}$

$$y - \frac{1}{3} = -\frac{1}{54}(x-9)$$

At time t minutes, the position of an object moving in a straight line is $p(t) = \frac{5t-9}{3+t}$ yards. **SCORE: _____ / 25 PTS**

[a] Find the average velocity of the object from time $t = 1$ to time $t = 5$. **Give the units of your answer.**

$$\frac{p(5)-p(1)}{5-1} = \frac{2 - -1}{4} = \frac{3}{4} \text{ YARDS/MINUTE}$$

[b] Find the instantaneous velocity of the object at time $t = 2$. **Give the units of your answer.**

$$\lim_{b \to 2} \frac{\frac{5b-9}{3+b} - \frac{1}{5}}{b-2}$$

$$= \lim_{b \to 2} \frac{5(5b-9)-(3+b)}{5(b-2)(3+b)}$$

$$= \lim_{b \to 2} \frac{\cancel{24b-48}\ 24}{5\cancel{(b-2)}(3+b)} = \frac{24}{25} \text{ YARDS/MINUTE}$$

If $\lim_{x\to 3} \frac{f(x)+7}{x-3} = -2$, find $\lim_{x\to 3} f(x)$ and prove that your answer is correct. **SCORE: _____ / 15 PTS**

$\lim_{x\to 3} \frac{f(x)+7}{x-3} \lim_{x\to 3}(x-3) = 2 \lim_{x\to 3}(x-3)$ SINCE $\lim_{x\to 3}(x-3)$ EXISTS (EQUALS 0)

$\lim_{x\to 3} \frac{f(x)+7}{x-3} \cdot (x-3) = 0$ SINCE $\lim_{x\to 3} \frac{f(x)+7}{x-3}$ ALSO EXISTS (GIVEN)

$\lim_{x\to 3} (f(x)+7) = 0$

$\lim_{x\to 3} (f(x)+7) + \lim_{x\to 3}(-7) = \lim_{x\to 3}(-7)$ SINCE $\lim_{x\to 3}(-7)$ EXISTS (EQUALS -7)

$\lim_{x\to 3} (f(x)+7-7) = -7$ IE. $\lim_{x\to 3} f(x) = -7$ SINCE $\lim_{x\to 3}(f(x)+7)$ ALSO EXISTS (SHOWN ABOVE)

Find the equations of all horizontal asymptotes of $f(x) = \dfrac{2e^{7x}+3}{15-8e^{7x}}$.

SCORE: _____ / 15 PTS

$$\lim_{x\to\infty} \frac{2e^{7x}+3}{15-8e^{7x}} \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2+3e^{-7x}}{15e^{-7x}-8}$$

$$= \frac{2+3(0)}{15(0)-8}$$

$$= -\frac{1}{4}$$

$$\lim_{x\to-\infty} \frac{2e^{7x}+3}{15-8e^{7x}}$$

$$= \frac{2(0)+3}{15-8(0)}$$

$$= \frac{1}{5}$$

$y = -\frac{1}{4}$, $y = \frac{1}{5}$